Department of Mathematics, Brooklyn College
Name $\qquad$ Section
Show all work and justify all your answers
PART I (55 points) : Answer all questions in this part.

1. (20 points)
(a) Find $\lim _{x \rightarrow \infty} \frac{3 x^{2}+4 x-1}{2 x^{2}-3}$.
(b) Find $d y / d x$ for each of the following:
i. $y=\left(e^{x}+4\right) \sqrt{3 x+2}$,
ii. $y=\frac{\ln (2 x)}{x^{2}+3}$,
iii. $y=\cos ^{x}(x)$,
iv. $y=\sec ^{4}(x)$.
2. (20 points) Shown is the graph of $f(x)=x^{4} / 4-2 x^{2}+1$.

(a) Determine and show on your graph all relative minima, relative maxima and points of inflection.
(b) Find intervals where the function is increasing, decreasing, concave up and concave down.
(c) Find the absolute maximum and the absolute minimum values of $f$ on the interval $[-1,3]$.
3. (15 points) Find each of the following integrals:
(a) $\int\left(x^{2}-x+5\right) \sqrt{x^{3}} d x$,
(b) $\int \sqrt{2 x+3} d x$,
(c) $\int_{0}^{\pi / 6} \cos ^{4}(2 x) \sin (2 x) d x$.

PART II (45 points) : Answer any three (3) of the four (4) questions in this part.

1. (15 points)
(a) Let $f(x)=1 /(3 x-1)$. Use the definition of derivative to find $f^{\prime}(1)$.
(b) Let $A$ be the area of a rectangle whose length $u$ and width $v$ are functions of time. At a certain time, the length is 15 feet and growing at the rate of 3 feet per second while the width is 7 feet and decreasing at the rate of 2 feet per second. Find the rate at which $A$ is changing at that time. Is the area increasing or decreasing at that time?
2. (15 points)
(a) Find the area between the curve $y=\sin (2 x)$ and the $x$-axis from $x=\pi / 6$ to $x=\pi / 3$.
(b) Find an equation of the tangent line to the curve $x^{2}+4 x y-y^{3}=1$ at the point $(1,2)$.

## 3. (15 points)

(a) A rectangular poster is to have an area of 200 square inches with 1 inch margin on the sides and a 2 inch margin at the top and at the bottom. Find the dimensions of the poster with the largest printed area.
(b) Find $\quad \lim _{x \rightarrow 0} \frac{\sin (4 x)}{\tan (7 x)}$.
4. (15 points) An object moves along a straight line. At time $t$ seconds its position (distance from the origin) is $s$ feet. Its acceleration $a$ at time $t$ is given by

$$
a(t)=6 t-7, \quad t \geq 0
$$

(a) Find its velocity $v$ at time $t$ given that $v(0)=2$.
(b) At what times is the velocity of the object 0 ? What is the velocity of the object at $t=3$ seconds?
(c) Find the position $s$ at time $t$, given that $s(0)=3$.

