

PRINT NAME. LAST: _____ FIRST: _____ OMITTED: _____

DEPARTMENT OF MATHEMATICS
BROOKLYN COLLEGE

FINAL EXAMINATION – FALL 2011
MATHEMATICS 3.3

SHOW ALL WORK. NO CREDIT WILL BE GIVEN UNLESS WORK IS SHOWN. ALL ANSWERS SHOULD BE GIVEN EXACTLY; DECIMAL APPROXIMATIONS WILL NOT BE ACCEPTED. DO ALL PROBLEMS IN PART I AND ANY FOUR PROBLEMS IN PART II.

PART I: *Do all problems in this part. (44 points)*

(16 pts) 1. Find dy/dx for each of the following:

(a) $y = \frac{(3x + 4)^8}{(x^2 + 2)^4}$

(b) $y = \ln(e^{3x} + 1) + e^{x^3 - 2}$

(c) $y = (\sin x)^{x^2+1}$

(d) $y = \sqrt{\sec(4x + 3)}$

(16 pts) 2. Find each of the following:

(a) $\int \frac{4x^7 - 6}{x^3} dx$

(b) $\int \frac{x+2}{x^2+4x} dx$

(c) $\int \frac{(\ln x)^4}{x} dx$

(d) $\int_0^{\pi/4} \cos^2(2x) \sin(2x) dx$

12 pts) 3. Consider the function

$$f(x) = x^3 + 6x^2 + 9x + 3.$$

(a) Find the intervals on which f is (i) increasing, (ii) decreasing, (iii) concave up, and (iv) concave down.

(b) Find the x -coordinates of the points where f has a (i) local maximum, a (ii) local minimum, or a (iii) point of inflection.

PA1 (c) Sketch the graph of f , clearly indicating the features identified in parts (a) and (b).

4. (a) Using the definition of derivative, find $f'(a)$ for $f(x) = \sqrt{x-1}$. (Obtaining $f'(x)$ by using the rules of differentiation will earn absolutely no credit.)

(b) Find the limit $\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x - 3}$.

PART II: Do any **four** of the five problems in this part. (14 points each, a total of 56 points)

4. (a) Using the definition of derivative, find $f'(x)$ for $f(x) = \sqrt{3 - 6x}$. (Obtaining $f'(x)$ by using the rules of differentiation will earn *absolutely no credit*.)

(b) A ship leaving port at 10 a.m. travels north with a speed of 10 miles per hour. Another ship, leaving the same port at 12 noon travels east with a speed of 30 miles per hour. How fast is the distance between the two

- (b) Find the limit $\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x - 3}$.

5. (a) Suppose $f(3) = -4$, $f'(3) = 3$, and $f''(x) = 6x - 8$. Find $f(x)$.

(b) A ship leaving port at 10 a.m. travels north with a speed of 10 miles per hour. Another ship, leaving the same port at 12 noon travels east with a speed of 20 miles per hour. How fast is the distance between the two ships increasing at 4 p.m.

6. (a) A ball is thrown upwards with an initial velocity of 16 feet/sec from the top of a 96 foot tall building. Assuming that the only force affecting the ball during travel is from gravity, which produces a downward acceleration of 32 ft/sec, find the velocity of the ball when it hits the ground.

- (b) Find the *places* of (i) absolute maximum and (ii) absolute minimum of the function $f(x) = 8x + 108x^{-2}$ on the interval $[1, 6]$.

7. (a) Find the equation of the tangent line to the curve $x^3y + y^2 + 7 = 0$ at the point $(2, -1)$.

(b) Find $\lim_{x \rightarrow +\infty} (\sqrt{4x^2 + 16x} - 2x)$.

8. (a) A box with a square base and open top must have a total surface area of 1200 cm^2 . What are the dimensions of such a box with the largest possible volume.

- (b) Find the area under the graph of $y = 1/\sqrt{x+3}$, above the x axis, and between the lines $x = 1$ and $x = 22$.